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Limitations of ASME PTC 10 in Accurately Evaluating Centrifugal Compressor Thermodynamic Performance

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ABSTRACT

The American Society of Mechanical Engineers (ASME) "Performance Test Code on Compressors and Exhausters," ASME PTC 10-1997, has served worldwide as the standard for determining centrifugal compressor thermodynamic performance during factory acceptance and field performance testing. The methodology applied in PTC 10 is based upon the original work of Schultz (1962) which utilizes a polytropic model of the compression process. Recent efforts associated with the selection and testing of compressors for high pressure carbon dioxide/hydrocarbon mixture reinjection compression services have revealed that the application of PTC 10 can result in significant errors in the estimation of compressor polytropic head and efficiency. The defined evaluation procedures use compressor suction and discharge pressures and temperatures along with gas composition to determine the thermodynamic properties necessary to derive the estimated head and efficiency. While it is recognized that these properties vary continuously from suction to discharge, average properties are used along with a correction factor proposed by Schultz (1962) to correct for variations in these properties along the actual compression path followed. Previous investigations by Mallen and Saville (1977), Huntington (1985), and Hundseid, Bakken and Helde (2006) have identified errors in some compressor applications and have proposed methods to address these issues, however, they have not fully identified the root cause nor the conditions under which these errors become significant.

The results of this effort will:

1. Illustrate those portions of the test code procedures that are subject to errors caused by unique process conditions.
2. Identify those gas conditions where the errors incurred in the application of the code are significant.
3. Propose an alternate evaluation method of test results and predictions that will reduce errors in the calculation of derived properties relevant to determination of compressor performance parameters.

Actual factory and field test data have been utilized where appropriate and available to support the theoretical principles developed in this study.

INTRODUCTION

The American Society of Mechanical Engineers performance test code for centrifugal compressors, ASME PTC 10-1997, and its International Standards Organization equivalent, ISO 5389-2005, serve as recognized standards for centrifugal compressor factory acceptance and field performance testing. These standards prescribe specific requirements and calculation methods for the thermodynamic performance testing of centrifugal compressors handling gases of various compositions and pressure and temperature conditions. The methods contained within these standards have resulted in the very accurate estimation of compressor performance parameters in a vast majority of cases, however, recent experiences with high pressure applications, particularly with mixtures containing high percentages of carbon dioxide, have revealed issues with the accuracy of predicted parameters including derived polytropic head and efficiency.

Calculation of the critical thermodynamic parameters necessary to evaluate compressor performance defined within both ASME PTC 10 and ISO 5389 are largely based upon the methodology developed by Schultz (1962) in his seminal paper covering the subject. Schultz (1962) developed methods to evaluate and predict compressor performance for a polytropic model based upon ideal and real gas relations. While there have been a limited number of critical evaluations of the accuracy of the methods developed by Schultz (1962) to date, they have substantially remained unchanged in the aforementioned industry standards and considered to be the most accurate available.

One of the first efforts to propose modifications to the methods developed by Schultz (1962) was presented in the technical paper written by Mallen and Saville (1977). Their basic premise was that the polytropic path function assumed by Schultz (1962) was only accurate at lower pressure levels. They proposed an alternate path function and accompanying analysis, but they admitted uncertainty as to whether their method was more accurate. Like Schultz (1962), some of the assumptions applied in Mallen and Saville's (1977) analysis were only applicable to ideal gas conditions. Polytropic head (or more correctly work) is a path dependent function which is influenced by actual variations in thermodynamic parameters along the path. These are normally not known and only the starting suction conditions and ending discharge conditions are available. One important concept suggested by Mallen and Saville (1977) was the numerical integration of the polytropic work function along the path by dividing it into a number of smaller segments to provide greater accuracy, although there were no guidelines or examples provided to demonstrate this method.

Nathoo and Gottenberg (1983) followed with essentially a review of the Schultz (1962) method, however, their intent was to highlight the importance of enhancing the accuracy of the analysis of compressors operating on hydrocarbon gas mixtures through accurate gas sampling and the proper application of equations of state to develop compositionally-based thermodynamic properties. While they also questioned the accuracy of the assumed polytropic path of the compressor work function, their conclusion was that it provided acceptable results in most practical applications. They both described and provided some examples of an incremental calculation method, including two examples from the Mallen and Saville (1977) paper, but their conclusion was that the method proposed by Schultz (1962) resulted in satisfactory accuracy.

Huntington (1985) also provided a relatively detailed, critical review of the procedures suggested by Schultz (1962) and Mallen and Saville (1977). His conclusions, however, differed from those of Nathoo and Gottenberg (1983) in that he found that errors inherent in both methods could be significant in certain conditions. Furthermore Huntington (1985) concluded that the method proposed by Mallen and Saville (1977) resulted in larger errors than those derived from the Schultz (1962) method. Huntington (1985) provided a two-segment, incremental calculation that defined a mid-path point which made the overall calculation more accurate. He also provided some examples where he compared his method against those of Schultz (1962) and Mallen and Saville (1977), showing improved accuracy validated through a more complex but undocumented numerical integration of the thermodynamic parameters.

The most recent work found addressing the subject is the paper authored by Hundseid, Bakken and Helde (2006). They questioned the precision of using an average value of the polytropic exponent and demonstrated deviations in results from method variations identified by Schultz (1962), although these variations are seldom used for testing and not widely promoted by either ASME PTC 10 or ISO 5389. A comparison of results obtained through these methods was made relative to a numerical integration for different cases, two of which were provided. Finally, an analysis of using different equations of state to derive the associated thermodynamic properties was provided.

While all of these evaluations identified weaknesses in the assumptions made by Schultz (1962) regarding the path followed in the compression process and derivation of the polytropic exponent used within the calculations, all neglected to fully explain the cause of the errors and the conditions under which such errors may become significant.

BACKGROUND

Calculation methods prescribed in ASME PTC 10-1997 and ISO 5389 are largely based upon the pioneering work of Schultz (1962) in his paper entitled "The Polytropic Analysis of Centrifugal Compressors" which presents a development of both isentropic and polytropic methods for compressor test evaluation, selection and design. While the isentropic analysis procedure presented by Schultz (1962) is generally easier to apply, the vast majority of compression equipment suppliers utilize the polytropic method, especially those supplying compressors for the process industries. This is due to the fact that the derived polytropic efficiency is theoretically constant for variations in suction conditions and compression ratio across a given machine. This is not the case for the isentropic method where efficiencies are subject to variation as applied pressures and pressure ratios are modified in a specific compressor. While the polytropic method is widely used in process compressor analysis, the isentropic method still finds predominant application in the air compression industry and gas turbine axial compressor performance among others.

The polytropic analysis method may simply be best described as the summation of incremental isentropic analysis steps as the pressure ratio across each step approaches a limit of unity. This will be an important concept to maintain as the following investigation is developed.

Performance test evaluation of a compressor requires only knowledge of stagnation (total) temperatures and pressures existing at the suction and discharge flanges of the machine and an accurate composition of the gas being handled. These parameters allow the calculation of compressor head (or the work applied to the fluid) and efficiency. The inclusion of gas mass flow rate permits the required power to be estimated. The addition of compressor operating rotational speed and either predicted or as-tested performance characteristics further enables the user to compare actual compressor operation against expected performance and provide an assessment of overall health or acceptability. The basis of a polytropic head calculation is developed by combining two fundamental relationships from thermodynamics. The first is the equation for work input to a fluid given as:

$$W = \int v dP$$

and the relationship for a polytropic process of an ideal gas which is given by:

$$Pv^n = constant$$

Substitution of the second relation into the first with some further manipulation and integration (a more complete derivation is provided in Appendix A) results in the two recognized relationships for polytropic head provided in ASME PTC 10:

$$W_p' = \left(\frac{n}{n-1}\right)[P_d v_d - P_s v_s]$$

$$= \left(\frac{n}{n-1}\right)\frac{Z_s R T_s}{MW}\left[\left(\frac{P_d}{P_s}\right)^{\left(\frac{n-1}{n}\right)} - 1\right]$$

Examination of the expressions above demonstrates that all but one of the parameters necessary to calculate the compressor head are available or derivable from the property measurements identified previously. The single parameter requiring further definition is the polytropic exponent, *n*, which can be estimated through manipulation of the relation for a polytropic process. An average, constant assumed value of *n* can be calculated from the following expression applied to a real gas:

$$n = \frac{ln\left(\frac{P_d}{P_s}\right)}{ln\left(\frac{v_s}{v_d}\right)} = \frac{ln\left(\frac{P_d}{P_s}\right)}{ln\left(\frac{P_d}{P_s}\frac{Z_s T_s}{Z_d T_d}\right)}$$

While many thermodynamic functions may be described as point functions with their values only related to the specific point independent thermodynamic parameters such as pressure and temperature, work is different. Work is a path dependent function with its ultimate value not only dependent upon the starting and finishing point parameters but also on the shape of the path between these endpoints. This path is influenced by the value of the polytropic exponent. Schultz (1962) recognized that this fact could impact the accuracy of the head predicted by the equation above and developed a correction factor to account for potential path differences. This correction factor was developed for an isentropic compression process and was assumed to be approximately the same and directly applicable to polytropic compression processes. It is estimated by the following relation:

$$f_S = \frac{h_{di} - h_s}{\left(\frac{n_i}{n_i - 1}\right)(P_d v_{di} - P_s v_s)}$$

The expected value of the Schultz correction factor, f_S, is near unity for the vast majority of applications. Since the factor is derived from an assumed isentropic process, the theoretical value should always be unity because the enthalpy increase and work applied to the gas should be equal since there is no increase in entropy, however, variations in the isentropic exponent, n_i, result in differences in the calculated absorbed work. The isentropic exponent is derived from a similar relationship to that of the polytropic exponent with the exception that the discharge specific volume is at isentropic conditions. It is theoretically equal to the specific heat ratio for an ideal gas. Significant differences in the isentropic exponent between suction and discharge conditions can result in errors by using an average value as is done in the above relationship.

Insertion of this correction factor into the equation for compression work results in the ASME PTC 10 modified equation for developed polytropic head:

$$W_p = f_S\left(\frac{n}{n-1}\right)[P_d v_d - P_s v_s]$$

$$= f_S\left(\frac{n}{n-1}\right)\frac{Z_s R T_s}{MW}\left[\left(\frac{P_d}{P_s}\right)^{\left(\frac{n-1}{n}\right)} - 1\right]$$

Polytropic efficiency is defined as the ratio of the compression work applied to the gas and the total energy absorbed from suction to discharge conditions. Total energy absorbed is reflected through the increase in enthalpy of the gas from suction to discharge. The difference between total energy absorbed and compression work applied to the gas is reflected by an increase in entropy of the gas. Polytropic efficiency therefore, is defined as:

$$\eta_p = \frac{W_p}{h_d - h_s}$$

Observations made over an extended period of time on a significant number of compressor performance tests have demonstrated that in the vast majority of cases the Schultz correction factor, f_S, is normally very close to a value of unity. This is particularly true for ASME PTC 10 Type 2 tests, however, in a limited number of cases the value has fallen

below 0.99, most notably observed during full load, Type 1 testing or on field tests. Since Type 2 tests are performed on relatively inert gases at low-pressure, near ideal conditions, it was postulated that relatively significant variations in the correction factor could be indicative of errors with the polytropic path assumption and inaccuracies in the calculated performance parameters. An incremental numerical integration of the compression path comparable to those described in the previous investigations was developed to explore the impact of the current method in accurately determining compressor performance parameters.

Dividing the total compression path into a large number of smaller segments with properties calculated for each segment should result in a more accurate value of compressor polytropic head and efficiency. The fundamental assumption of constant efficiency along a polytropic path is a necessary condition applied to the numerical integration of the compression path. This can be expressed for each increment of the integration as:

$$\eta_p = \frac{\Delta W_{pi}}{\Delta h_i}$$

An incremental calculation method was developed in accordance with the following summary. A more complete logic flow diagram is provided in Appendix B.

1. Set an incremental compression ratio, Rc_i, relative to the number of segments, n_{incr}, assumed from the following relation:
$$Rc_i = \sqrt[n_{incr}]{\frac{P_d}{P_s}}$$
2. Assume a polytropic efficiency, η_p, initially equal to the overall calculation value.
3. Using the segment suction temperature and pressure along with the segment discharge pressure, calculate the incremental polytropic head, enthalpy change and polytropic efficiency using the isentropic segment discharge temperature as an initial estimate.
4. Iterate on the segment discharge temperature until the segment calculated polytropic efficiency is equal to the overall polytropic efficiency.
5. Move to next segment, setting segment suction conditions equal to previous segment discharge conditions. Repeat steps 2 through 4 for each segment.
6. Once calculation is completed for final segment, compare final segment discharge temperature to actual overall measured discharge temperature. If they are not equal, modify the assumed polytropic efficiency and repeat steps 2 through 6 until the two temperatures are equal.
7. Calculate revised values of overall polytropic head and enthalpy change by summing the values obtained for each segment. Validate the assumed efficiency by taking the ratio of polytropic head to enthalpy change.

This methodology was applied to a number of different cases where the Schultz (1962) correction factor derived from the normal overall calculation deviated relatively significantly from unity. It was also applied to a select number of other cases where the correction factor approached unity for independent verification of its utility. The number of increments required to provide increased accuracy was determined through the calculation of the Schultz (1962) correction factor for individual segments. These values were required to be very close to unity, otherwise the number of increments was increased to provide greater accuracy.

INCREMENTAL CALCULATION METHOD

An incremental calculation procedure was developed in compliance with the previously described algorithm. Thermodynamic properties were evaluated using the GERG-2008 equation of state (Kunz and Wagner, 2012) as implemented in the National Institute of Standards and Technology (NIST) REFPROP v.9 (Lemmon, et. al., 2010) computer software application. An initial number of calculation increments was chosen to be 10 considering maximum typical compression ratios provided in actual compressors and the resulting ratio across each increment. Sensitivity runs were completed on selected examples using a larger number of increments but additional accuracy was not considered to be significant with a larger number of increments. One quantitative measure used to validate the accuracy of the incremental calculation was identified as the value of the Schultz (1962) factor obtained for each increment. A value of the Schultz (1962) factor was required to be greater than 0.999 which represented a minimal expected deviation between calculated and actual paths. Accordingly, all examples included in this evaluation were calculated using 10 increments along the compression path to provide computational consistency between the different conditions.

Three hypothetical cases were created to investigate the variation in the polytropic exponent across a compression path and the validity of the assumptions of constant polytropic exponent and constant value of the polytropic process function. The first case, identified as Case AW, was chosen to be a near ideal gas application with lower pressures and operation well into the superheat region. Pure methane was chosen as the gas to be compressed from suction conditions of 100 psia and 100F to a discharge of 300 psia and 290F. The discharge temperature was chosen to reflect a compression efficiency of approximately 82% which is consistent with current designs. Figure 1 demonstrates the variation in the polytropic exponent across the incremental compression path compared to the average calculated polytropic exponent for Case AW.

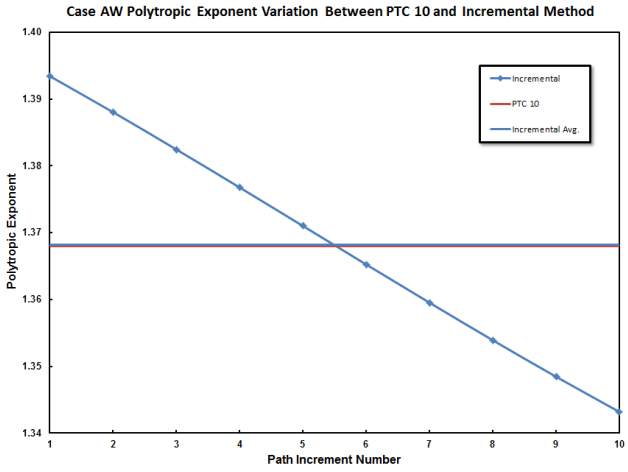


Figure 1. Polytropic Exponent Variation for Case AW

While the preceding figure clearly shows that the value of the polytropic exponent in this case varies across the compression path, a closer look at the actual magnitude of this variation is only about 2% above and below the average. Another way to look at the impact of polytropic exponent variation along the actual compression path is to compare the pressure-specific volume product at each increment calculated with both the average and point values of polytropic exponent used to calculate the specific volume. An additional measure is the variation in the polytropic process function (Pv^n) along the compression path. These alternate methods to evaluate the accuracy of the determination of the compression work are provided for Case AW in Figure 2. The results of this analysis, along with the other two examples presented in this section, have been normalized to more readily allow comparison between the different cases. The pressures and specific volumes have been normalized to suction conditions, whereas the polytropic process function has been normalized to the average values.

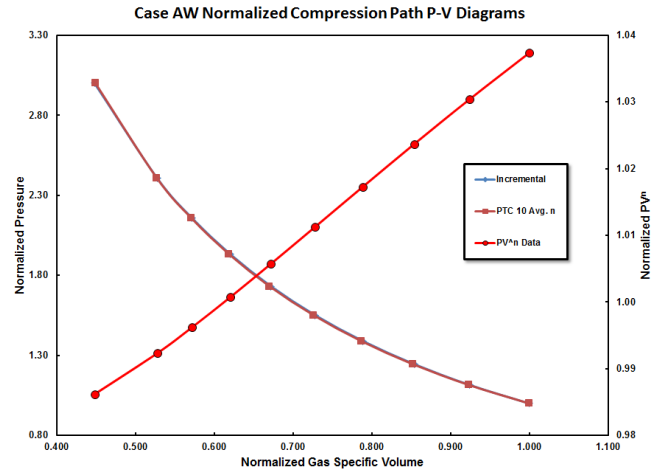


Figure 2. Case AW PV Diagrams

It is apparent from a review of this diagram that the pressure-specific volume curves derived from both the PTC 10 average polytropic exponent essentially overlays the same characteristic derived from the point specific, incremental value. Since the area under the pressure-specific volume curve represents the compression work applied to the gas and the fact that the curves derived from the two methods are effectively the same, it is obvious that the two methods in this case yield very similar values of polytropic head and efficiency. This analysis also provides further confirmation of the accuracy of the Schultz (1962) method under these conditions. An alternative method used to estimate the accuracy of the PTC 10 calculations is available through an examination of the variation in the value of the polytropic process function along the path. It is assumed to be a constant in the development of the polytropic work function. Although the above figure shows that the polytropic work function does vary over the compression path, an actual comparison of the values of the function from suction to discharge shows a ratio of only 1.052. This very small difference in value of the polytropic process function further validates the accuracy of the assumption of a constant value for this specific case.

A second, hypothetical compression application was developed to compare and contrast the Schultz (1962) method and incremental method under different and more challenging conditions. Since a compromise in accuracy of the Schultz (1962) method that prompted this investigation was suspected with a mixed hydrocarbon and carbon dioxide mixture, a similar gas mixture was selected. This second case, designated as Case AX, was composed of a 70% carbon dioxide, 30% methane mixture. Suction conditions were established in the superheat region below the critical pressure but above the critical temperature. Discharge conditions were selected in the dense-phase region above the critical pressure and at a compression ratio consistent with the first example. The suction pressure was chosen to be 800 psia and the suction temperature was set at 100F. Discharge conditions were set at 2400 psia and 290F with the discharge temperature chosen to maintain an efficiency of approximately 81%. The variation in polytropic exponent for this case is provided in Figure 3.

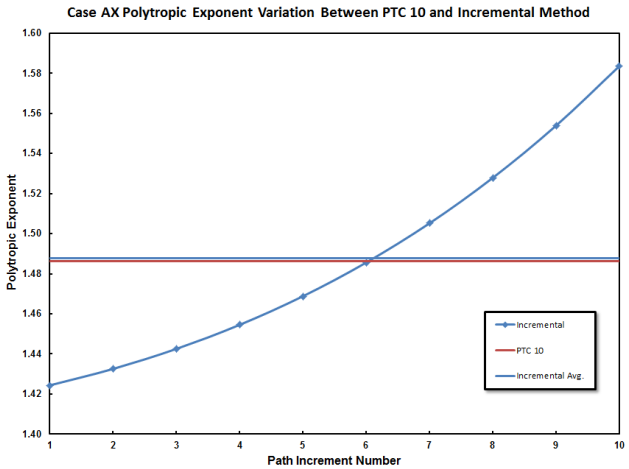


Figure 3. Polytropic Exponent Variation for Case AX

This case also shows a variation in polytropic exponent along the compression path, but a comparison against Case AW shows that the polytropic exponent rises from suction to discharge in this case while it drops in Case AW. The reason

for this difference will become obvious in subsequent analysis, however, it should be noted that the direction in the variation of polytropic exponent is not as important as the change in magnitude. A numerical evaluation of the variation in polytropic exponent results in a variation of approximately 4% below the average at the suction to more than 6.5% above the average at discharge conditions. This approximate 11% variation in polytropic exponent is considerably greater than the 4% realized in the first example. A similar evaluation of the pressure-specific volume characteristic and the polytropic process function for the second case, Case AX, is provided in Figure 4.

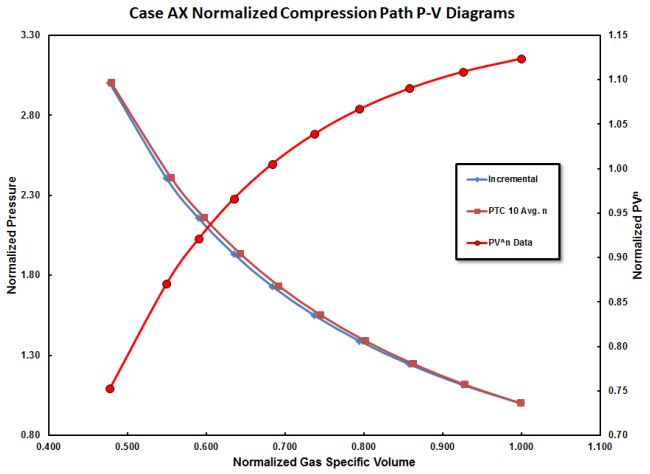


Figure 4. Case AX PV Diagrams

There is a noticeable difference between the pressure-specific volume characteristics derived from the Schultz (1962) and incremental calculation methods for this case. Since it has been previously noted that the area under the pressure-specific volume curves represents the work, or polytropic head, related to the compression process, a deviation in calculated polytropic head between the two methods is expected. A comparable evaluation of the variation in polytropic process function value along the compression path shows a more significant change than the first case with a ratio of over 1.49. While it is evident that the variation in calculated parameters is small, there is nevertheless a difference between the methods as the gas conditions become more challenging.

A third case, designated as Case AY, was developed to provide an anticipated more challenging set of conditions for compressor evaluation. The gas composition of 70% carbon dioxide and 30% methane was maintained from the Case AX. Suction conditions were moved from the superheat region into the dense phase region with a suction pressure of 2000 psia above the mixture pseudo-critical pressure and a suction temperature of 100F which is approximately 15% above the mixture pseudo-critical temperature. The discharge pressure was set at 6000 psia to maintain an equivalent pressure ratio and a discharge temperature of 230F to maintain a similar polytropic efficiency of approximately 81%. Figure 5 provides a comparison of the variation in polytropic exponent along the compression path against the average value for Case AY.

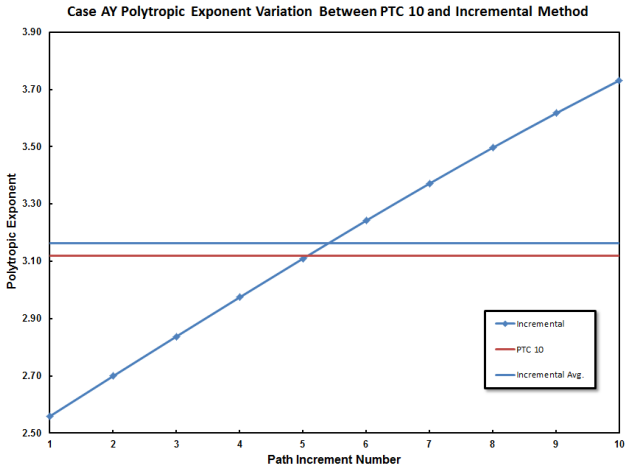


Figure 5. Polytropic Exponent Variation for Case AY

Consistent with the previous two cases, the value of the polytropic exponent varies along the compression path and increases from suction to discharge as with the second case. The degree of variation in the polytropic exponent is greater in this case when compared against the other two. This variation ranges from approximately 18% below the average at suction conditions to over 20% above the average at discharge resulting in a 45% variation across the path. The impact of more substantial variation in polytropic exponent in this case on the pressure-specific volume characteristic and the polytropic process function value is provided in Figure 6.

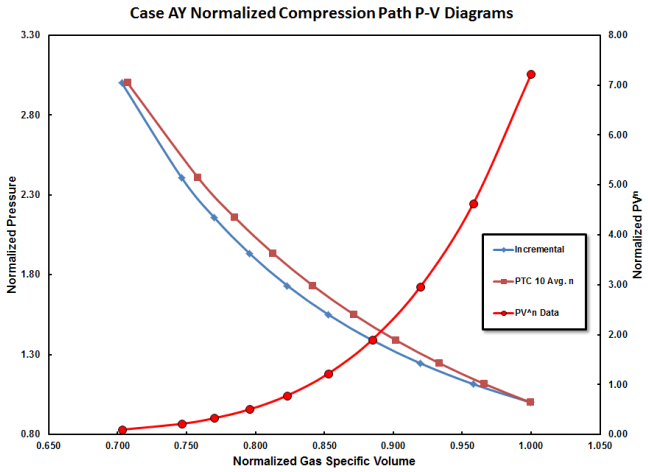


Figure 6. Case AY PV Diagrams

The deviation in the pressure-specific volume characteristics is more pronounced in this case than the previous two. Directionally this leads to the conclusion of a greater departure in calculated polytropic head between the Schultz (1962) method and the incremental calculation. There is also a larger difference in the polytropic process function along the compression path in this case when compared against the other two evaluated. In this case, the ratio of this value across the path is 47.62.

A summary of the selected data resulting from these analyses is provided in Table 1 to allow a more precise comparison of the results. An obvious conclusion that can be

drawn from these analyses is that variations in polytropic exponent increase as gas conditions deviate from ideal gas conditions. These deviations may compromise the ability of the current methods within ASME PTC 10 to accurately evaluate and predict compressor performance under increasingly non-ideal conditions. In such cases, an alternate calculation method may need to be applied to improve accuracy, particularly when considering that there continues to be demands to reduce performance tolerances on compressor performance coupled with more challenging operating conditions evolving in some applications.

Table 1. Polytropic Exponent Evaluation Data

Case	AW	AX	AY
Gas Composition	100% C1	70% CO2 / 30% C1	70% CO2 / 30% C1
Suction Pressure, psia	100	800	2000
Suction Temperature, F	100	100	100
Discharge Pressure, psia	300	2400	6000
Discharge Temperature, F	290	290	230
Polytropic Exponent Results			
PTC 10 Average Value	1.3680	1.4862	3.1181
Suction Conditions Exponent Value	1.3934	1.4244	2.5587
Discharge Conditions Exponent Value	1.3432	1.5836	3.7318
Suction Deviation, %	+1.86	-4.16	-17.94
Discharge Deviation, %	-1.81	+6.55	+19.68
Polytropic Process Function (Pv^n)			
Suction Conditions Function Value	622.564	64.507	0.867
Discharge Conditions Function Value	591.849	43.205	0.018
Polytropic Process Function Ratio	1.052	1.493	47.620

POLYTROPIC EXPONENT EVALUATION

The variability of the polytropic exponent and its potential influence on the accuracy of the polytropic head and efficiency has been demonstrated for a limited number of cases. Analysis of compressor performance by dividing the overall path into a number of segments of smaller pressure ratio has revealed that there can be a significant deviation in the polytropic exponent over the entire path from suction to discharge conditions. The existing method calculates an average value of polytropic exponent by evaluating known conditions at the suction and discharge. While relatively minor deviations in this value from average do not significantly affect the accuracy of calculated head and efficiency, larger deviations may compromise the validity of these calculated values.

An evaluation of the polytropic exponent, *n*, and its dependence upon pressure and temperature was developed. Schultz (1962) provided a very detailed derivation of the polytropic path exponent which resulted in the following relation:

$$n = \frac{1}{Y - m(1 + X)}$$

This relatively simple equation is actually quite complex in that all of its variables are derivative properties of the gas compressibility factor, Z, which in turn is related to the specific equation of state employed in the analysis. Schultz (1962) provided a generalized derivation of these specific variables. The first, X, is defined as:

$$X = \frac{T}{v}\left(\frac{\partial v}{\partial T}\right)_P - 1$$

$$= \frac{T}{Z}\left(\frac{\partial Z}{\partial T}\right)_P$$

The parameter, $\frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P$, is known as the coefficient of thermal expansion and is analogous to the familiar property of the same name for solids. Its equivalent expressed as a function of the compressibility factor, Z, can be derived from the real gas equation. The second primary term, Y, in the polytropic exponent relation is defined as:

$$Y = -\frac{P}{v}\left(\frac{\partial v}{\partial P}\right)_T$$

$$= 1 - \frac{P}{Z}\left(\frac{\partial Z}{\partial P}\right)_T$$

This term, $-\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_T$, is known as the isothermal compressibility and is related to the property of the bulk modulus in liquids. In a similar fashion, it's derivation in terms of the compressibility factor can be developed from the real gas equation. The third and final term contained within the equation for the polytropic exponent is obtained from the following relation:

$$m = \frac{ZR}{C_P}\left(\frac{1}{\eta_P} + X\right)$$

It is evident that this term is a function of the polytropic efficiency and thermodynamic properties that are functions of the pressure and temperature; namely the compressibility factor, the constant pressure specific heat, and the coefficient of thermal expansion. When combined the individual terms of the polytropic exponent equation allow the calculation of its value at a single point if the pressure, temperature and polytropic efficiency are known at that point. The thermodynamic properties involved may be obtained from one of a number of equations of state. Accuracy of these values is highly dependent upon the ability of the chosen equation of state to predict the actual gas or gas mixture PVT properties.

Point values of the polytropic exponent were derived for three significantly different gases at two specific pressures per gas over a range of temperatures. The three gases investigated were hydrogen, carbon dioxide and R134a (1,1,1,2 – tetraflouroethane) representing a significant molecular weight range from 2.016 for hydrogen to 102.032 for R134a. This wide

molecular weight range covers most industrially significant gases. Results of this analysis are provided in Figure 7. The properties utilized to develop this figure were derived from the GERG-2008 equation of state (Kunz and Wagner, 2012) implemented within the National Institute of Standards and Technology (NIST) REFPROP v.9 software application (Lemmon, et. al., 2010). GERG is recognized as being one of the most accurate equations of state available to predict thermodynamic properties. Similar analyses could be developed using different equations of state with anticipated small deviations, however, the general trends of the behavior of the polytropic exponent are expected to be comparable. It should also be noted that the polytropic efficiency assumed in this analysis was 80%. Variations in efficiency will result in differences, but they are not substantial within the normal expected range of polytropic efficiencies realized in actual compressors.

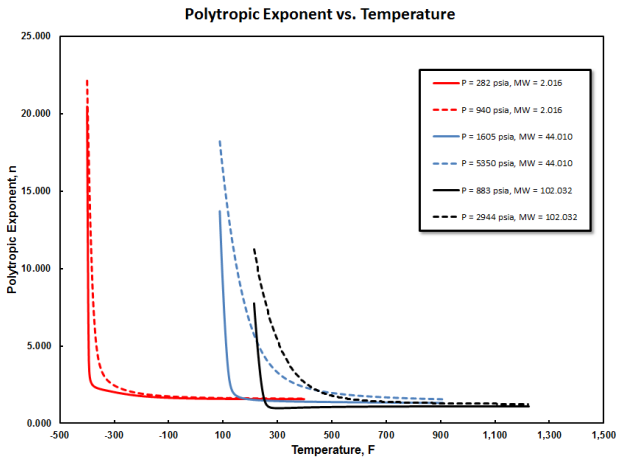


Figure 7. Polytropic Exponent for Selected Gases

It is evident from this figure that at the lower portion of the temperature range for each individual gas, there is a significant variation in the polytropic exponent with small changes in temperature. This is characterized by the near vertical curve shape. At higher temperatures, the characteristics approach a horizontal line with very little change in polytropic exponent over a large range of temperature difference. While the location of these characteristics and their transitions vary significantly between gases over the temperature range provided in the figure, they demonstrate similar tendencies.

The Principle of Corresponding States is an important relationship associated with the physical and thermodynamic properties of gases. It is based upon the concept that different gases behave in a similar fashion when normalized relative to some reference properties. These reference properties are the critical pressure, *Pc*, and the critical temperature, *Tc*, which are unique for each individual pure gas component. Gas properties are normalized by taking the ratio of the pressure and temperature at a point of interest relative to the critical properties and are designated as the reduced pressure, $Pr = P/Pc$, and reduced temperature, $Tr = T/Tc$, of the gas. This also applies to gas mixtures where pseudo-critical pressures and temperatures can be determined from a mixing rule combined with individual component critical properties. The most widely used mixing rule is that proposed by Kay which is simply the summation of the products of the individual gas component mole fraction and its critical pressure or temperature, respectively.

When the Principle of Corresponding States is applied to the data provided in Figure 7, a more consistent behavior is reflected between the three gases representing a significant range of molecular weights. This is demonstrated in Figure 8. While the absolute values of the polytropic exponents differ between the gases at low values of reduced temperature, all three gases exhibit consistent behavior with near vertical characteristics at reduced temperatures below 1.3. As the pressure is increased (as is reflected by the higher reduced pressure characteristic), the slope of the polytropic exponent at these lower reduced temperatures appears to decrease. Values of the polytropic exponent asymptotically approach constant values at reduced temperatures above 1.7 across the reduced pressure range provided. It is evident that these constant values of polytropic exponent approached are different between the three gases examined. This is due to differences in the molecular structure of the three gases and is similar to differences in the specific heat ratio existing at ideal conditions.

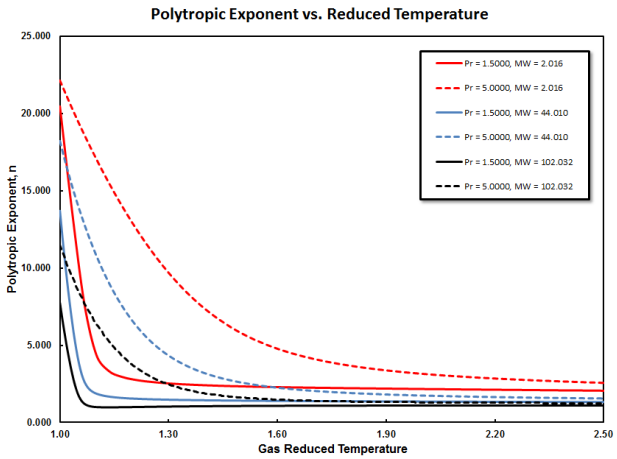


Figure 8. Polytropic Exponent in Terms of Reduced Properties

A compression process superimposed on this diagram would involve moving from a point of lower reduced pressure and temperature at suction conditions to one of a higher reduced pressure and temperature at discharge. At low values of reduced temperature, this could result in a significant variation of the polytropic exponent across the path. This is not the case in the higher reduced temperature region of the figure, where a variation in polytropic exponent is minimal from suction to discharge. While this is a significant observation for the data provided, it must be noted that this data has been provided at conditions only above the critical pressure. This limited region of the overall phase diagram for a single component gas is designated as the dense phase region and is characterized as an area where substances can behave as a liquid or gas. Operation of compressors in the dense phase region is industrially significant with many applications such as high pressure injection machines.

An evaluation of the polytropic exponent at lower reduced

pressures was also completed and the results of this analysis are provided in Figure 9. This area of the phase diagram is commonly referred to as the superheat region with the lower reduced temperature boundary coincident with the vapor pressure of the single component substance where any further reduction in reduced temperature will result in crossing into the two-phase region with the formation of liquid. For multi-component mixtures, this boundary is known as the dew point.

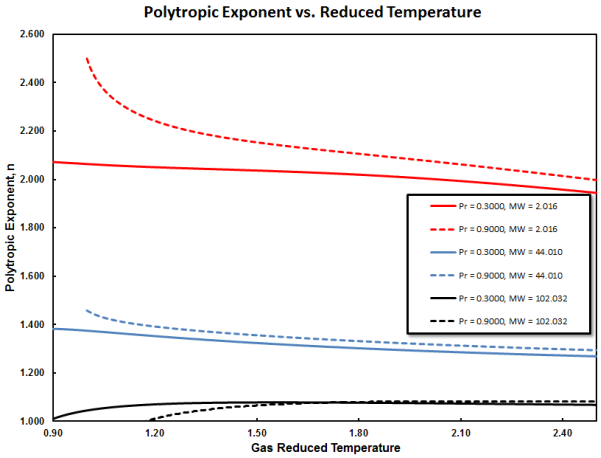


Figure 9. Polytropic Exponents in Superheat Region

Variations in the polytropic exponent in the superheat region across a range of reduced pressures are minimal, particularly at higher values of reduced temperature. A compression process occurring in this region will therefore result in only a slight variation of polytropic exponent and the assumption of an average value across the path incurs a minimal error in derived polytropic head and efficiency.

Carbon dioxide will be utilized to more fully illustrate the polytropic exponent behavior of a gas in both the dense phase and superheat regions. It is a valuable gas to use as an example due to its unique properties. The critical point of carbon dioxide (*Pc* = 1069 psia and *Tc* = 87.76 F) is located at conditions that exist in a wide variety of industrial applications that operate in both regions of interest. Recent activity in the areas of carbon sequestration and miscible gas injection for enhanced oil recovery has increased the utilization of carbon dioxide across a wide range of pressures. Coupled with the critical temperature in a range commonly existing in the environment, compressor applications handling carbon dioxide or significant amounts of carbon dioxide in gas mixtures can operate in both the superheat and dense phase regions. Figure 10 provides a comparison of polytropic exponent behavior across a wide range of reduced pressures and temperatures. The minimum reduced pressure evaluated was 0.3 and the maximum 5.0. As noted previously, there is only a slight variation in the polytropic exponent for operation in the superheat region where the reduced pressure is less than 1.0. Operation in the dense phase region where the reduced pressure is greater than 1.0 results in comparable behavior at reduced temperatures above 1.65. Under these conditions variations in polytropic exponent is minimal across an arbitrary path of compression from a lower pressure to a higher pressure. Potential compression paths into the dense phase region at reduced temperatures less than 1.65, however, can result in significant variations in the polytropic exponent.

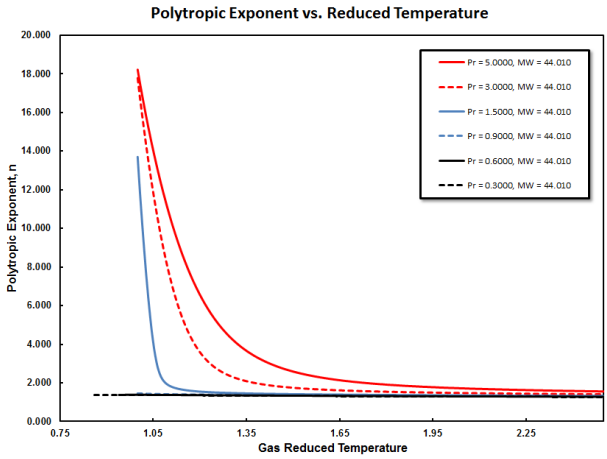


Figure 10. Polytropic Exponent Characteristics for Carbon Dioxide

The results of this analysis and acknowledgment that the Principle of Corresponding States applies across a vast majority of gases and gas mixtures, particularly those commonly found in industrial processes, allows the formulation of guidelines concerning the accuracy of the existing methodology utilized for determination of polytropic head and efficiency in both the ASME and ISO standards. Compressor performance evaluation of machines operating in the superheat region, that is with reduced pressures less than unity, and machines operating in the dense phase region at reduced temperatures across the compression path greater than 1.6 are highly likely to be of acceptable accuracy utilizing existing methods. Evaluation of machines operating in the dense phase region at reduced pressures less than 1.6 and machines with either suction or discharge conditions existing in the same region may incur significant errors. This is primarily due to the fact that the existing methods utilize an average value of the polytropic exponent across the compression path without significant deviation from that average value. It is obvious that this may not be true when suction and/or discharge conditions are above a reduced pressure of 1.0 and below a reduced temperature of 1.6. Several examples illustrating this concept will be provided in subsequent analysis. An alternative method of compressor performance evaluation may be warranted when these conditions exist.

REVISED CALCULATION METHOD – THE POLYTROPIC HEAD CORRECTION FACTOR

The previous analyses have demonstrated the limitations of the application of a constant, average value of polytropic exponent in accurately predicting the polytropic work of a compression process. This was validated through two separate analyses. The first involved dividing the overall compression ratio into a larger number of smaller, equal paths of compression ratio that when summed duplicated the overall compression path. Results from this analysis applied to a

number of different cases revealed that significant changes in polytropic exponent could occur in some situations that would compromise the accuracy of the calculated polytropic work. A second analysis examined the variation in polytropic exponent based upon thermodynamic relations across a range of reduced pressure and temperature conditions applicable across a wide range of single component gases and gas mixtures. This exercise demonstrated that the polytropic exponent could vary significantly across a compression path under specific conditions, primarily to those occurring above a reduced pressure of unity and below a reduced temperature of 1.6. These two different approaches both showed that variations in the polytropic exponent could negatively impact the accuracy of the derived polytropic work and efficiency.

Schultz (1962) also recognized that there could be variations across any given polytropic compression path. In an attempt to address these inaccuracies, he developed a correction factor based upon an isentropic compression path model. This correction factor is simply the ratio of the isentropic enthalpy difference to the calculated isentropic head. Since the value of this factor is theoretically valid for a comparison of the consistency of an isentropic compression path, Schultz (1962) postulated that it might also accurately correct inaccuracies in the relation for polytropic head. The basis of Schultz's (1962) evaluation is first derived from the application of one of Maxwell's Relations for thermodynamics. The derivative form of a combination of the First and Second Laws of Thermodynamics is provided by the familiar relation found in many textbooks:

$$dh = Tds + vdP$$

It becomes obvious from this equation that for an isentropic process where the change in entropy is zero, the compressor isentropic work is simply equal to the change in enthalpy, or:

$$\int dh = \int vdP$$

$$\Delta h_i = W_i$$

Stated in other terms, the path dependent value of isentropic work of a compression process is exactly equal to the difference between the isentropic discharge and suction enthalpies, both point functions independent of the path followed. When this result is compared to the more familiar equation for the isentropic head, the Schultz (1962) correction factor is derived:

$$f_S = \frac{h_{di} - h_s}{\left(\frac{n_i}{n_i - 1}\right)(P_d v_{di} - P_s v_s)}$$

Inspection of this relationship reveals that all properties with the exception of the isentropic exponent are thermodynamic point functions, directly measurable or derived from an equation of state. If the value of the isentropic exponent is a constant, the derived correction factor is unity. This is only true for an ideal gas where the gas specific heat ratio is constant. Deviations in the correction factor from unity, therefore, demonstrate variation in the isentropic exponent across the compression path. Larger deviations in correction factor from unity denote more significant deviations in isentropic exponent from suction to isentropic discharge conditions. Schultz (1962) reasoned that this factor should also be directly applicable to a polytropic compression process.

The incremental analysis method previously described was applied to a number of different actual compressor test results and design points. There was close agreement between the incremental calculation and the Schultz (1962) method applied with the correction factor for gas conditions near ideal, however, the more operating conditions deviated from ideal the greater the difference was realized between the two methods in some instances. A significant indicator of these deviations appeared to be correlated to the value of the correction factor. As it deviated increasingly from a value of unity, the greater was the likelihood that the two methods of analysis provided different results. This observation brings into question the accuracy and validity of applying the Schultz (1962) correction factor in all situations. The use of the incremental calculation method provides a more accurate estimation of the head and efficiency, but it is recognized that it is more difficult to implement in all situations even with the increased calculation capacity of modern computer technology.

Another examination of the previously listed Maxwell relation after some rearrangement and integration provides the following equation:

$$W = \int vdP = \int dh - \int Tds$$

This form of the relationship shows that the work path function can be expressed as the difference between two point functions; the integral of the enthalpy and the integral of the product of the temperature and the entropy of the gas. The enthalpy integral simplifies to the difference between the discharge and suction enthalpy of the compression process, however, integration of the second term is not as straightforward. Entropy is actually a function of both the pressure and temperature of the gas. While it is possible to integrate this term with complex relations for the entropy specific to each equation of state that might be employed to describe the gas behavior, a simple approximation is given by:

$$\int Tds \cong T_{avg}(s_d - s_s)$$

The average temperature proposed is simply the arithmetic mean value of the temperature from suction to discharge. Use of the log mean temperature was also evaluated but it was concluded that this provided less accurate results than the arithmetic mean for all of the examples examined. Although this number may appear to vary significantly, it must be understood that this is actually the arithmetic average of the absolute temperatures from suction to discharge. The addition of the substantial correction to convert measured temperature to absolute diminishes the percentage change in temperature from suction to discharge conditions and results in a relatively constant value of temperature.

It is therefore possible to propose a new correction factor as an alternative to the long-standing Schultz (1962) correction factor to potentially further improve the accuracy of derived values of polytropic head and efficiency for any given compressor application. This new correction factor, proposed to be referenced as the polytropic correction factor, is of the form:

$$f_p = \frac{(h_d - h_s) - T_{avg}(s_d - s_s)}{\left(\frac{n}{n-1}\right)[P_d v_d - P_s v_s]}$$

$$= \frac{(h_d - h_s) - T_{avg}(s_d - s_s)}{W_p'}$$

The resulting modified expression for the determination of the polytropic head for a compression process is provided by:

$$W_p = f_p W_p' = f_p \left(\frac{n}{n-1}\right)[P_d v_d - P_s v_s]$$

It obviously follows from the above equation that the polytropic work can simply be found from taking the difference between the enthalpy differential and the product of the average temperature and entropy difference. While this is possible and essentially gives the same result, calculation of the revised correction factor is consistent with the existing methodology inherent in the industry standards and likely easier to adopt. It should also be noted that calculation of the revised correction factor is more straightforward than the original one which requires an iterative calculation to determine the isentropic discharge conditions. The revised correction factor is determined from suction and discharge pressures and temperatures, and the derived properties of specific volume, enthalpy, entropy and polytropic exponent. Ultimately, the integration of the temperature-entropy function at suction and discharge conditions will be able to provide an exact solution, however, the proposed correction factor yields very accurate results and can more readily be incorporated into existing code calculations.

A method was developed to validate the improved accuracy of the proposed polytropic correction factor relative to the existing Schultz (1962) correction factor. While the limited examples provided thus far and a number of other case runs were completed comparing the resulting polytropic head and efficiency derived from suction and discharge conditions from a number of both actual and imagined applications, a more rigorous development was considered necessary to demonstrate improved accuracy. Forty-eight additional example cases were evaluated covering gases from pure methane and carbon dioxide to light and heavy hydrocarbon mixtures influenced by actual compression applications that received ASME Type 1 factory tests where deviations in calculated performance were suspected. In each of these cases, a six stage compressor model was constructed from the given inlet pressure and temperature conditions. An overall developed head and efficiency assumed for each model was distributed equally among the stages. Using a ten step increment for each stage, expected discharge pressure and temperature were calculated. This essentially provided an incremental prediction calculation of discharge conditions based upon sixty equal increments of head and efficiency.

Once this was complete, an equivalent "test" calculation was developed based upon the established suction and discharge conditions using the ten step incremental calculation defined herein, the existing Schultz (1962) method defined within ASME PTC 10, and the revised PTC 10 method replacing the Schultz (1962) correction factor with the proposed polytropic correction factor. Percentage error between the designated polytropic head and efficiency and that deduced from the three above defined test evaluation methods were computed. In all cases, the difference between the designated head and that resulting from the ten step incremental method was less than .05%. This provided assurance that the incremental test evaluation method based upon ten, equal pressure ratio steps resulted in acceptable accuracy since the comparison against a sixty step prediction calculation provided such close agreement. The ten step incremental test calculation was then set as the baseline for an accurate determination of polytropic head and efficiency based upon given suction and discharge conditions of pressure and temperature for a defined gas mixture.

A comparison was then completed between the ten step incremental method and the ASME PTC 10 calculation with both the original Schultz correction factor and the proposed polytropic correction factor. The percentage error in calculated polytropic head was then compared between the methods using the two different correction factors. The results are provided in Figure 11. It should be noted that differences in derived polytropic efficiency will be similar due to the fact that the efficiency is the ratio of head to enthalpy difference with the enthalpy difference being the same in both calculations. Effectively, a more accurate determination of polytropic head also results in a more accurate estimate of efficiency.

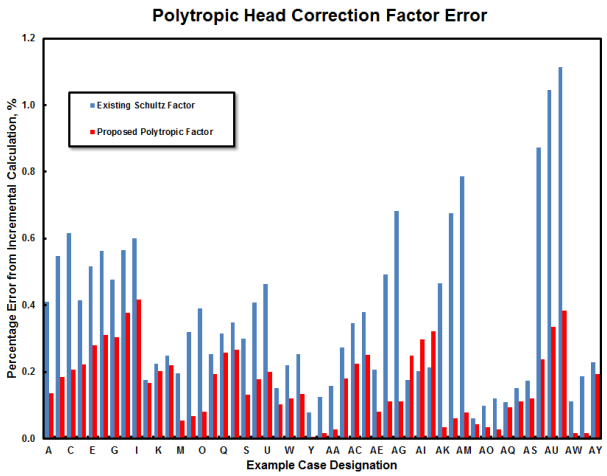


Figure 11. Correction Factor Accuracy Comparison

It is evident from this comparison that the proposed polytropic correction factor provides improved accuracy in the vast majority of cases evaluated. This is particularly true as the conditions deviate from ideal conditions. Other comparisons nearer to ideal conditions completed but not included here also show similar or better accuracy utilizing the proposed polytropic correction factor. This leads us to conclude that improved accuracy in the calculation of polytropic head and efficiency across the wide range of industrially relevant

applications is possible though the replacement of the existing Schultz (1962) correction factor with the polytropic correction factor. Appendix C provides the pressure and temperature conditions, gas compositions and derived values of polytropic head and efficiency for each of the different methods representing the forty eight test points provided in Figure 11 and can be referenced for independent analyses.

CONCLUSIONS

The existing method to estimate compressor polytropic performance as originally developed by Schultz (1962) and employed in the ASME PTC 10 and ISO 5389 standards has been evaluated for its overall accuracy. It has been found that under certain, defined conditions, the accuracy of the results derived from this method can incur significant error in predicting compressor polytropic head and efficiency. This potential impact on accuracy becomes more prevalent as gas conditions deviate from ideal conditions. The following actions were completed in this evaluation that supports these conclusions:

- An incremental calculation method was introduced which maintains the assumption of constant polytropic efficiency across the entire compression path. This method closely approximates an actual integration of the path-dependent function and provides a more accurate evaluation of the polytropic head and efficiency than the current method developed by Schultz (1962) and endorsed by both ASME PTC 10 and ISO 5389. While the results derived from this incremental method agree closely with the results of the existing code calculations in many cases, there are a number of examples identified in this study and previously by several other investigators where deviations between the two methods become significant. Although it is possible to apply the incremental method to these calculations going forward, it is a more complex operation that requires substantial iteration and represents a more complicated procedure even utilizing current computer capabilities.
- Application of the incremental method and detailed evaluation of the results demonstrated that in isolated cases the variation in polytropic exponent along the compression path could be significant. In these cases, the use of the current method and an average value of polytropic exponent may compromise the accuracy of the results. A point value analysis of the polytropic exponent utilizing the Principle of Corresponding States provided further insight into the behavior of this parameter across a wide range of temperatures and pressures for a specific gas or gas mixture. Variation in the polytropic exponent under ideal gas conditions and compressors operating below the critical pressure of the gas was found to be limited across any arbitrary path. In these cases, the existing methods will provide acceptable accuracy of derived head and efficiency. When suction and/or discharge pressures exceed a reduced pressure of 1.0 and the reduced temperature falls below a value of 1.6, results obtained from the existing code calculation methods may include a significant error that could impact conclusions regarding the acceptability of a given compressor's performance.
- Deviations in a polytropic compression path were anticipated by Schultz (1962) through an analysis of an isentropic path under the same conditions. A correction factor was developed for the isentropic path variation which was theorized to be comparable and applicable to a polytropic path. The incremental path calculations completed in this study revealed that use of the existing Schultz (1962) correction factor could result in significant error in some actual cases. Examination of the Maxwell relations for thermodynamics applied to an arbitrary compression process illustrated that the work of compression could be obtained through an evaluation of the change in enthalpy and an entropy function, both of which are point functions and independent of the actual compression path. While it is possible to integrate the entropy function to obtain an exact value of the compression work, this may be quite complicated and dependent upon the specific equation of state employed to predict the gas thermodynamic properties. An approximate method was developed to determine this parameter and incorporated through the application of a revised polytropic correction factor. Substitution of this correction factor in place of the existing Schultz (1962) correction factor shows very close agreement between the incremental calculation method and current polytropic method. This is valid across the entire range of possible compressor applications from ideal gas to operation into and within the dense phase region.

A revised, more universally accurate method to evaluate compressor thermodynamic performance has been developed and demonstrated through this effort. This method may be utilized to evaluate factory acceptance and field testing, prediction of compressor operating parameters during initial design, and prediction of actual operating conditions from low pressure, inert gas test results. Although exact solutions are possible from an integration of the governing equations, this could be complex and specific to different equations of state utilized to predict gas thermodynamic properties. An approximate method has been developed which shows close agreement to an integration applied to a number of examples. This approximate method is also capable of being manipulated to provide a new correction factor which can be used in place of the existing Schultz (1962) correction factor to improve the accuracy of polytropic head and efficiency calculations in their current form. The impact of applying this new polytropic correction factor by manufacturers, users and others involved in evaluating compressor performance will be minimal since all of the parameters required are available in the current analysis method and actually more directly derived than the Schultz (1962) factor. Adoption of this revised method into the existing industry performance test codes is recommended to improve the accuracy of the calculations involved.

NOMENCLATURE

Variables:

Cp	= gas constant pressure specific heat
f_p	= polytropic work correction factor
f_S	= Schultz work correction factor
h	= enthalpy
m	= polytropic exponent function
MW	= gas (mixture) molecular weight
n	= polytropic exponent
n_i	= isentropic exponent
n_{incr}	= number of incremental integration steps
P	= pressure
R	= universal gas constant
Rc_i	= incremental compression ratio
S	= entropy
T	= temperature
v	= gas specific volume
W	= work
W_p'	= uncorrected polytropic work (head)
W_p	= corrected polytropic work (head)
X	= coefficient of thermal expansion function
Y	= coefficient of isothermal compressibility function
Z	= real gas compressibility factor
η_p	= polytropic efficiency

Subscripts:

avg	= arithmetic average
d	= discharge conditions
di	= isentropic process discharge conditions
i	= isentropic process
p	= polytropic process
s	= suction conditions

APPENDIX A –
DEVELOPMENT OF POLYTROPIC HEAD (WORK) EQUATIONS FOR COMPRESSORS

The differential form of the First Law of Thermodynamics gives the work term as:

$$W = \int v dP$$

Furthermore, a polytropic process for an ideal gas is defined by:

$$Pv^n = constant$$

which can be rearranged to the following form:

$$v = \frac{constant}{P^{1/n}}$$

Now, substituting this result for the specific volume into the equation for polytropic work:

$$W_P' = constant \int \frac{dP}{P^{1/n}} = constant \int (P^{-1/n})\, dP$$

evaluation of the integral from suction to discharge conditions gives:

$$W_P' = constant \left[\frac{P^{\left(\frac{-1}{n}+1\right)}}{\left(\frac{-1}{n}+1\right)} \right]_s^d = constant \left[\frac{P^{(n-1)/n}}{\left(\frac{n-1}{n}\right)} \right]_s^d$$

but, since:

$$P^{-1/n} = \frac{v}{constant}$$

and, upon substitution into the previous equation results in one of the recognized forms of the polytropic head equation:

$$W_P' = \left(\frac{n}{n-1}\right)[P_d v_d - P_s v_s]$$

However, returning to the integrated form of the equation above:

$$W_P' = constant \left[\frac{P^{\left(\frac{-1}{n}+1\right)}}{\left(\frac{-1}{n}+1\right)} \right]_s^d = constant \left[\frac{P^{(n-1)/n}}{\left(\frac{n-1}{n}\right)} \right]_s^d$$

expanding:

$$W_P' = constant \left(\frac{n}{n-1}\right)\left[{P_d}^{(n-1)/n} - {P_s}^{(n-1)/n}\right]$$

and rearranging:

$$W_P' = constant \left(\frac{n}{n-1}\right){P_s}^{(n-1)/n}\left[\left(\frac{P_d}{P_s}\right)^{(n-1)/n} - 1\right]$$

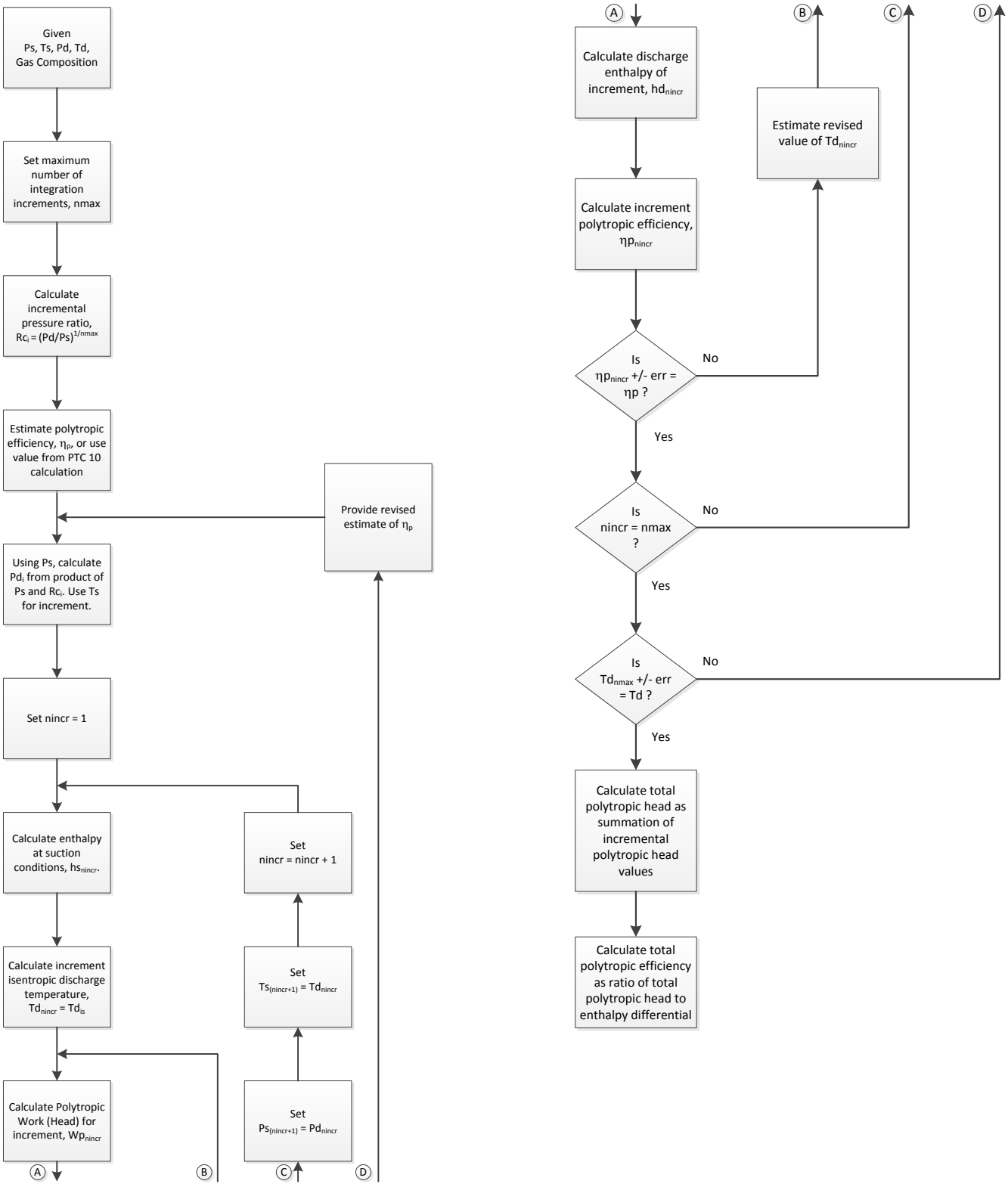
but since it can be shown that:

$${P_s}^{(n-1)/n} = \frac{P_s v_s}{constant} = \frac{Z_s R T_s}{constant(MW)}$$

upon substitution, yields the second familiar form of the polytropic head equation:

$$W_P' = \left(\frac{n}{n-1}\right)\frac{Z_s R T_s}{MW}\left[\left(\frac{P_d}{P_s}\right)^{(n-1)/n} - 1\right]$$

APPENDIX B –
LOGIC DIAGRAM FOR INCREMENTAL CALCULATION METHOD



APPENDIX C –
EVALUATION DATA

Compressor Performance Parameters

COMPARISON TABLE

			Given						PTC 10-1997		Proposed Method		Incremental		Deviation From Given Conditions		
Example	Gas	MW	Poly. Head (ft-lbf/lbm)	Poly. Eff.	P1 (psia)	P2 (psia)	T1 (F)	T2 (F)	Poly. Head (ft-lbf/lbm)	Poly. Eff.	Poly. Head (ft-lbf/lbm)	Poly. Eff.	Poly. Head (ft-lbf/lbm)	Poly. Eff.	PTC 10 (%)	Proposed (%)	Incr (%)
A	CO2-Methane	30.03	34794	0.82	1500.0	5364.3	90.0	291.3	34658	0.8169	34848	0.8214	34801	0.8203	-0.39	0.16	0.02
B	CO2-Methane	30.03	43983	0.82	1500.0	6738.8	90.0	331.0	43756	0.8127	44078	0.8217	43997	0.8202	-0.52	0.22	0.03
C	CO2-Methane	30.03	48738	0.82	1500.0	7496.2	90.0	350.0	48452	0.8152	48853	0.8219	48752	0.8202	-0.59	0.24	0.03
D	CO2-Methane	30.03	34794	0.82	1700.0	6685.9	70.0	259.2	34657	0.8167	34878	0.8219	34801	0.8201	-0.39	0.24	0.02
E	CO2-Methane	30.03	43983	0.82	1700.0	8359.2	70.0	294.5	43771	0.8160	44121	0.8226	43998	0.8203	-0.48	0.31	0.03
F	CO2-Methane	30.03	48738	0.82	1700.0	9266.9	70.0	311.5	48478	0.8157	48903	0.8228	48752	0.8203	-0.53	0.34	0.03
G	CO2-HCmix	36.49	34794	0.82	1649.7	8189.7	101.2	316.2	34638	0.8164	34910	0.8228	34804	0.8203	-0.45	0.33	0.03
H	CO2-HCmix	36.49	43983	0.82	1649.7	10388.1	101.2	354.0	43752	0.8157	44167	0.8234	44001	0.8203	-0.53	0.42	0.04
I	CO2-HCmix	36.49	48738	0.82	1649.7	11577.4	101.2	372.1	48462	0.8154	48958	0.8237	48755	0.8203	-0.57	0.45	0.03
J	CO2-HCmix	26.23	34794	0.82	3138.6	8615.6	98.5	223.4	34735	0.8187	34854	0.8215	34796	0.8201	-0.17	0.17	0.01
K	CO2-HCmix	26.23	43983	0.82	3138.6	10273.4	98.5	248.8	43890	0.8182	44078	0.8217	43989	0.8201	-0.21	0.22	0.01
L	CO2-HCmix	26.23	48738	0.82	3138.6	11157.3	98.5	261.2	48621	0.8180	48849	0.8219	48742	0.8201	-0.24	0.23	0.01
M	HC mix	24.29	34794	0.82	868.8	2405.3	124.9	267.1	34730	0.8185	34817	0.8206	34798	0.8201	-0.18	0.07	0.01
N	HC mix	24.29	43983	0.82	868.8	3002.5	124.9	298.8	43854	0.8175	44025	0.8207	43995	0.8201	-0.29	0.10	0.03
O	HC mix	24.29	48738	0.82	868.8	3342.7	124.9	314.1	48559	0.8170	48788	0.8208	48749	0.8202	-0.37	0.10	0.02
P	HC mix	24.29	34794	0.82	1891.3	5438.2	120.0	238.3	34712	0.8181	34867	0.8217	34800	0.8201	-0.24	0.21	0.02
Q	HC mix	24.29	43983	0.82	1891.3	6576.0	120.0	260.5	43856	0.8176	44108	0.8222	43995	0.8202	-0.29	0.28	0.03
R	HC mix	24.29	48738	0.82	1891.3	7188.6	120.0	271.1	48580	0.8173	48880	0.8223	48750	0.8201	-0.32	0.29	0.02
S	HC mix	23.38	34794	0.82	1319.0	3736.5	110.0	247.3	34697	0.8176	34847	0.8212	34801	0.8201	-0.28	0.15	0.02
T	HC mix	23.38	43983	0.82	1319.0	4589.4	110.0	274.6	43817	0.8169	44075	0.8217	43997	0.8202	-0.38	0.21	0.03
U	HC mix	23.38	48738	0.82	1319.0	5059.7	110.0	287.6	48526	0.8165	48850	0.8219	48752	0.8203	-0.43	0.23	0.03
V	HC mix	18.28	34794	0.82	2138.0	4485.2	110.0	222.8	34743	0.8187	34832	0.8208	34796	0.8200	-0.15	0.11	0.01
W	HC mix	18.28	43983	0.82	2138.0	5235.1	110.0	247.4	43893	0.8183	44043	0.8211	43990	0.8201	-0.20	0.14	0.02
X	HC mix	18.28	48738	0.82	2138.0	5641.5	110.0	259.5	48620	0.8179	48808	0.8211	48743	0.8200	-0.24	0.14	0.01
Y	HC mix	20.73	34794	0.82	537.0	1288.7	59.0	197.9	34768	0.8194	34797	0.8201	34795	0.8201	-0.07	0.01	0.00
Z	HC mix	20.73	43983	0.82	537.0	1570.1	59.0	232.0	43932	0.8190	43980	0.8199	43987	0.8200	-0.12	-0.01	0.01
AA	HC mix	20.73	48738	0.82	537.0	1730.9	59.0	249.2	48663	0.8187	48727	0.8197	48740	0.8200	-0.15	-0.02	0.00
AB	HC mix	20.73	34794	0.82	1754.8	5275.5	20.0	149.7	34704	0.8179	34862	0.8216	34799	0.8201	-0.26	0.20	0.01
AC	HC mix	20.73	43983	0.82	1754.8	6392.3	20.0	175.2	43841	0.8174	44092	0.8220	43993	0.8202	-0.32	0.25	0.02
AD	HC mix	20.73	48738	0.82	1754.8	6992.7	20.0	187.5	48561	0.8170	48868	0.8221	48746	0.8201	-0.36	0.27	0.02
AE	HC mix	38.7	34794	0.82	335.0	1695.8	100.0	311.7	34726	0.8183	34770	0.8193	34798	0.8200	-0.20	-0.07	0.01
AF	HC mix	38.7	43983	0.82	335.0	2416.6	100.0	361.4	43780	0.8162	43948	0.8193	43997	0.8203	-0.46	-0.08	0.03
AG	HC mix	38.7	48738	0.82	335.0	2862.0	100.0	385.1	48423	0.8147	48701	0.8194	48756	0.8203	-0.65	-0.08	0.04
AH	HC mix	35.05	34794	0.82	2071.0	8172.4	160.0	272.0	34737	0.8186	34884	0.8220	34798	0.8200	-0.16	0.26	0.01
AI	HC mix	35.05	43983	0.82	2071.0	10025.7	160.0	291.9	43904	0.8184	44124	0.8225	43993	0.8200	-0.18	0.32	0.02
AJ	HC mix	35.05	48738	0.82	2071.0	11010.8	160.0	301.5	48640	0.8184	48902	0.8228	48745	0.8201	-0.20	0.34	0.01
AK	CO2-Methane	35.62	34794	0.82	1000.0	4264.3	100.0	349.2	34639	0.8164	34813	0.8205	34801	0.8202	-0.45	0.05	0.02
AL	CO2-Methane	35.62	43983	0.82	1000.0	5571.9	100.0	400.5	43703	0.8146	44027	0.8207	44000	0.8202	-0.64	0.10	0.04
AM	CO2-Methane	35.62	48738	0.82	1000.0	6312.6	100.0	425.0	48372	0.8139	48794	0.8209	48756	0.8203	-0.75	0.11	0.04
AN	Methane	16.04	34794	0.82	1000.0	1891.1	100.0	209.4	34773	0.8195	34809	0.8204	34794	0.8200	-0.06	0.04	0.00
AO	Methane	16.04	43983	0.82	1000.0	2190.6	100.0	236.3	43944	0.8193	44002	0.8203	43987	0.8201	-0.09	0.04	0.01
AP	Methane	16.04	48738	0.82	1000.0	2356.4	100.0	249.9	48681	0.8191	48754	0.8203	48740	0.8200	-0.12	0.03	0.00
AQ	Methane	16.04	34794	0.82	3000.0	5596.5	100.0	194.4	34758	0.8193	34829	0.8210	34796	0.8200	-0.10	0.10	0.01
AR	Methane	16.04	43983	0.82	3000.0	6381.1	100.0	215.6	43921	0.8188	44037	0.8210	43988	0.8201	-0.14	0.12	0.01
AS	Methane	16.04	48738	0.82	3000.0	6800.8	100.0	226.1	48656	0.8185	48800	0.8209	48741	0.8200	-0.17	0.13	0.01
AT	CO2	44.01	34794	0.82	1000.0	6923.9	100.0	407.3	34508	0.8133	34895	0.8224	34812	0.8205	-0.82	0.29	0.05
AU	CO2	44.01	43983	0.82	1000.0	9222.4	100.0	459.9	43554	0.8120	44162	0.8233	44014	0.8205	-0.98	0.41	0.07
AV	CO2	44.01	48738	0.82	1000.0	10492.7	100.0	484.6	48229	0.8114	48959	0.8237	48772	0.8206	-1.04	0.45	0.07
AW	Methane	16.04			100.0	300.0	100.0	290.0	68320	0.8151	68386	0.8159	68397	0.8160	-0.11	-0.02	
AX	CO2-Methane	35.62			800.0	2400.0	100.0	290.0	25638	0.8093	25682	0.8107	25686	0.8108	-0.19	-0.02	
AY	CO2-Methane	35.62			2000.0	6000.0	100.0	230.0	20520	0.8094	20607	0.8128	20567	0.8112	-0.23	0.19	

Gas Compositions

		EXAMPLE CASE DESIGNATION											
		A-B-C D-E-F	G-H-I	J-K-L	M-N-O P-Q-R	S-T-U	V-W-X	Y-Z-AA AA-AB-AC	AE-AF-AG	AH-AI-AJ	AK-AL-AM AX-AY	AN-AO-AP AQ-AR-AS AW	AT-AU-AV
Methane (C1)	16.043	50	25.6274	60.4842	68.8671	74.2574	85.8644	80.4000	18.5790	30.2940	30	100	
Ethane (C2)	30.070		2.2871	5.5619	11.9957	7.4107	11.5319	5.3500	1.5920	3.7480			
Propane (C3)	44.097		0.5691	1.5369	10.2964	9.7710	0.4501	1.6900	37.9660	43.5330			
n-Butane (n-C4)	58.122		0.0001	0.0010	2.7541	3.9704	0.0001	0.1700	0.3071	0.2180			
i-Butane (i-C4)	58.122		0.0001	0.0010	3.1432	1.5802	0.0001	0.1100	0.0884	0.2220			
n-Pentane (n-C5)	72.149		0.0001	0.0010	0.5563	0.5301		0.0100	0.0884				
i-Pentane (i-C5)	72.149		0.0001	0.0010	0.8927	0.6701		0.0200	0.0134				
n-Hexane (n-C6)	86.175		0.0175	0.0551		0.1800							
Carbon Dioxide (CO2)	44.010	50	71.2356	31.6646	1.2468	1.3901	1.4903	11.6400	41.3000	21.5860	70		100
Nitrogen (N2)	28.013		0.2629	0.6933	0.2477	0.2400	0.6601	0.6100	0.0657	0.3990			
Hydrogen Sulfide (H2S)	34.081						0.0030						

Calculated Mole Weight (lbm/lbmol):	30.026	36.490	26.233	24.287	23.376	18.283	20.730	38.699	35.051	35.620	16.043	44.010
Critical Pressure (psia)	872.21	956.76	798.91	665.48	665.55	681.66	718.86	816.11	734.82	951.86	673.08	1071.3
Critical Temperature (°F)	-14.3	35.5	-35.7	-25.1	-37.1	-88.9	-75.5	95.8	77.2	26.6	-116.4	87.9

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